FURTHER MATHS TERM BY TERM CURRICULUM

Year 12

Term	Teacher 1 (Further Core Mathematics) delivered via 3 x 70-minute lesson per week	Teacher 2 (Decision Mathematics & Further Mechanics) delivered via 1 x 70 minute lesson per week
1	**FC1.1 Complex Numbers** • Be able to solve any quadratic equation with real coefficients. • Be able to add, subtract and multiply complex numbers in the form $x+$ iy with x and y real. • Understand and use the terms 'real part' and 'imaginary part'. • Be able to use and interpret Argand diagrams. • Be able to convert between the Cartesian form and the modulus-argument form of a complex number. • Be able to multiply and divide complex numbers in modulus-argument form. • Be able to construct and interpret simple loci in the Argand diagram such as $\lvert z-a\rvert > r$ and $\arg(z-a)=\theta$.	**D1.1 Algorithms** • Understand what an algorithm is. • Be able to trace an algorithm in the form of a flow chart. • Be able to trace an algorithm given as instructions written in text. • Know how to determine the output of an algorithm and how it links to the input. • Be able to determine the order of a given algorithm and standard network problems. • Know how to apply a bubble sort algorithm to a list of numbers or words. • Know how to apply the quick sort algorithm to a list of numbers or words, clearly identifying the pivots used for each pass. • Be able to identify the number of comparisons and swaps used in a given pass. • Be able to identify size, efficiency and order of an algorithm and use them to make predictions. • Know how to solve bin packing problems using full bin, first fit, and first fit decreasing algorithms, and understand their strengths and weaknesses. • Know the meaning of the vocabulary used in graph theory e.g. degree of a vertex, isomorphic graphs, walks, paths and cycles. • Be familiar with different types of graph e.g. complete, planar, isomorphic, simple, connected. • Understand graphs represented in matrix form. • Be familiar with k notation. • Know the definition of a tree.

		Be able to determine if a graph is Eulerian, semi-Eulerian or neither, and find Eulerian cycles.
	FC1.6 Matrices • Be able to find the dimension of a matrix. • Be able to add and subtract matrices of the same dimension. • Be able to multiply a matrix by a scalar. • Be able to multiply conformable matrices. • Be able to calculate determinants of 2×2 and 3×3 matrices. • Understand and use singular and non-singular matrices. • Be able to know the properties of inverse matrices. • Be able to calculate the inverse of non-singular 2×2 and 3×3 matrices. • Be able to use matrices and their inverses to solve linear simultaneous equations, including three linear simultaneous equations in three variables. • Be able to interpret geometrically the solution and failure of solution of three simultaneous linear equations. • Be able to use matrices to represent 2D rotations, reflections, enlargements and translations. • Understand and use zero and identity matrices. • Be able to use matrix products to represent combinations of transformations. • Be able to use matrices to represent linear transformations in three dimensions. • Be able to use inverse matrices to reverse the effect of a linear transformation. • Be able to use the determinant of a matrix to determine the area scale factor of a transformation. • Be able to find invariant points and lines for a linear transformation.	**D1.2 Algorithms on Graphs (Part 1)** • Understand the meaning of a minimum spanning tree. • Be able to apply Kruskal's algorithm to a network to find the minimum spanning tree. • Be able to apply Prims algorithm to a network to find the minimum spanning tree. • Be able to apply Prim's algorithm to a distance matrix to find the minimum spanning tree. • Be able to apply Dijkstra's algorithm to find the shortest path between two vertices in a network. • Be able to trace back through a network to be able to find the route corresponding to the shortest path. • Be able to consider modifications to an original shortest path problem, for example by dealing with multiple start points or a different end point.
2	**FC1.3 Roots of Polynomials** • Understand and use the complex conjugate of a complex number. • Be able to divide two complex numbers by using the complex conjugate of the denominator. • Know that non-real roots of polynomial equations with real coefficients occur in conjugate pairs.	**D1.4 Algorithms on Graphs (Part 2)** • Be able to find all the shortest paths between all the pairs of vertices using Floyd's algorithm.

	• Be able to solve cubic or quartic equations with real coefficients.	
	FC1.4 Series • Be able to use sigma notation. • Understand and use formulae for the sums of integers, squares and cubes. • Be able to use known formulae to sum more complex series.	**D1.7 Linear Programming** • Know how to formulate a linear programming problem from a real-life problem (write inequalities from worded questions). • Be able to form an appropriate objective function to maximise or minimise. • Know how to represent a linear programming problem graphically and identify the feasible region. • Be able to solve linear programming problems to find a maximum or minimum. • Be able to interpret solutions in the context of the original real-life problem.
3	**FC1.5 Transforming Polynomials** • Understand and use the relationship between roots and coefficients of polynomial equations up to quartic equations. • Be able to form a polynomial equation whose roots are a linear transformation of the roots of a given polynomial equation (of at least cubic degree).	**D1.9 Critical Path Analysis** • Be able to model a project by an activity network from a precedence table. • Be able to complete a precedence table from a given network. • Understand the use of dummies. • Know how to carry out a forward pass and backward pass using early and late event times. • Be able to interpret and use dummies. • Be able to identify critical activities and critical paths. • Know how to determine the total float of activities. • Be able to construct and interpret Gantt (cascade) charts.
	FC1.7 Proof • Be able to obtain a proof for the summation of a series, using induction. • Be able to use proof by induction to prove that an expression is divisible by a certain integer. • Be able to use mathematical induction to prove general statements involving matrix multiplication.	
4	**FC1.8 Vectors** • Know how to find the vector equation of a line in both two and three dimensions.	**FM1.1 Momentum & Impulse (Part 1)** • Understand the definitions, derivation, and units of momentum and impulse.

	• Understand and use the Cartesian forms of an equation of a straight line in three dimensions. • Understand and use the vector and Cartesian forms of the equation of a plane. • Be able to find the scalar product of two vectors. • Be able to check whether vectors are perpendicular by using the scalar product. • Be able to use the scalar product to express the equation of a plane. • Be able to use the scalar product to calculate the angle between two lines. • Be able to use the scalar product to calculate the angle between two planes. • Be able to use the scalar product to calculate the angle between a line and a plane. • Be able to find the points of intersection of lines and planes which meet. • Be able to calculate the perpendicular distance between two lines. • Be able to calculate the perpendicular distance from a point to a line or to a plane.	• Understand what happens to the momentum of a sphere as a result of a collision. • Be able to use the principle of conservation of momentum applied to direct collisions in 1-dimension. **FM1.3 Work, Energy & Power** • Understand the derivation, units and definitions of work and energy. • Be able to define kinetic energy (KE). • Understand that work done on a body moving in a horizontal plane is the change in kinetic energy. • Understand the concept of gravitational potential energy (GPE). • Be able to include GPE when applying the work-energy principle. • Know the conditions for conservation of mechanical energy. • Be able to solve problems involving work and energy. • Understand that power in watts is the rate of doing work. • Be able to calculate the power (P) of a vehicle with a tractive (driving) force F, moving with velocity v. • Be able to use the formula $P = Fv$ in problem solving.
5	**FC1.4 Volumes of Revolution** • Be able to derive formulae for and calculate volumes of revolution about both the x and y-axes. • Be able to solve modelling problems which involve volumes of revolution.	**FM1.5 Elastic Collisions in One Dimension** • Be able to express the 'compressibility', 'bounciness' or 'elasticity' of an object by a value called the coefficient of restitution (e). • Know that $0 \leq e \leq 1$ [and that e = 0 means inelastic and e =1 means perfectly elastic]. • Know and be able to use Newton's (experimental) law of restitution for direct impacts of elastic spheres. • Be able to calculate the change in kinetic energy due to an impact. • Be able to solve problems of the following types involving elastic impacts:

		a) successive collisions between pairs of spheres (horizontal motion). b) bouncing ball (off a horizontal elastic plane). c) successive collisions including two spheres and sphere against a wall. d) determination of the number of collisions or deriving the possible range of *e*.
6	**Revision for AS exams** **AS exams**	**Revision for AS exams** **AS exams**
	Paper 1: Further Core Mathematics 1 50%, 1 hour 40 mins, 80 marks	Only Year 12 content assessed
	Paper 2: Decision Mathematics 1 & Further Mechanics 1 50%, 1 hour 40 mins, 80 marks	Only Year 12 content assessed Section A (25% of AS Level) Section B (25% of AS Level)

Year 13

Term	Teacher 1 (Further Core Mathematics) delivered via 3 x 70-minute lesson per week	Teacher 2 (Decision Mathematics & Further Mechanics) delivered via 1 x 70 minute lesson per week
1	**FC2.1 Complex Numbers** • Be able to multiply and divide complex numbers in modulus-argument and exponential form. • Know and use cosine and sine in terms of the exponential form. • Understand, remember and be able to use de Moivre's theorem: $z^n = r^n e^{in\vartheta} = r^n(\sin n\vartheta + i \cos n\vartheta)$. • Be able to derive multiple angle formulae/expressions e.g. $\cos 3\vartheta$ in terms of powers of $\cos\vartheta$, and $\sin^3\vartheta$ in terms of multiple angles of $\sin\vartheta$. • Be able to apply de Moivre's theorem to sum a geometric series. • Know how to solve completely equations of the form $z^n - a - ib = 0$, giving special attention to cases where $a = 1$, $b = 0$.	**D1.3 The Planarity Algorithm** • Be able to apply the planarity algorithm for planar graphs. • Be able to determine if a graph contains a Hamiltonian cycle.
	FC2.5 Hyperbolic Functions • Know the definitions of $\sinh x$, $\cosh x$ and $\tanh x$ including their domains and ranges. • Be able to sketch graphs of the hyperbolic functions. • Be able to differentiate and integrate the hyperbolic functions and know the standard results. • Understand and be able to use the inverse hyperbolic functions including domains and ranges. • Be able to derive, use and know the logarithmic forms of the inverse hyperbolic functions.	**D1.5 Route Inspection** • Be able to determine whether a graph is traversable. • Be able to apply an algorithm to solve the route inspection problem; • Find a route by inspection. • Understand the importance of the order of vertices of the graph in finding a route.
2	**FC2.4 Polar Coordinates** • Understand and be able to use polar coordinates and be able to convert between polar and Cartesian coordinates.	**D1.6 The Travelling Salesperson Problem** • Understand the travelling salesman problem and that there is no simple algorithm to solve it for complex networks.

	• Know how to sketch standard polar curves. • Be able to find tangents parallel and perpendicular to the initial line. • Be able to find (compound) areas under polar graphs using the formula $\frac{1}{2}\int r^2 \mathrm{d}\theta$.	• Be able to use the nearest neighbour algorithm to find upper bounds for the problem. • Be able to find lower bounds for a problem. • Understand that not all upper and lower bounds give a solution to the problem. • Know how to identify the best upper and lower bounds. • Be able to solve the travelling salesman problem and interpret this solution in the context of the problem.
	FC2.2 Series • Be able to use the method of differences to sum simple finite series. • Be able to find and use higher derivatives of functions. • Know how to express functions as an infinite series in ascending powers using Maclaurin's expansion. • Be able to find the series expansion of composite functions.	**D1.8 The Simplex Algorithm** • Understand and use slack, surplus and artificial variables. • Be able to use slack variables to write inequality constraints as equations. • Know how to rewrite LP problems so that each equation contains all the variables *x, y, s,* and *t*. • Be able to put the information in an initial tableau. • Be able to find the pivot and use it to form a new tableau. • Be able to identify if a tableau satisfies the optimality condition. • Know how to use slack and surplus variables. • Understand and be able to use artificial variables. • Be able to use the two-stage simplex algorithm. • Be able to use the Big-M method. • Be able to relate the solution to the original problem.
3	**FC2.3 Further Calculus** • Know how to deal with infinity as a limit of a definite integral. • Be able to integrate functions across limits which include values when the function is undefined i.e. deal with discontinuous integrands. • Understand and be able to evaluate the mean value of a function. • Be able to integrate functions which can be split into partial fractions up to denominators with quadratic factors. • Be able to differentiate inverse trigonometric functions such as $\frac{1}{2}\arctan x^2$. • Know how to integrate functions of the form $(a^2 - x^2)^{-\frac{1}{2}}$ and $(a^2 + x^2)^{-1}$ and be able to choose trigonometric substitutions to	**D1.10 Resource Histograms** • Be able to draw and interpret resource histograms. • Be able to level resource histograms. • Be able to construct a scheduling diagram. • Be able to interpret and modify schedules to meet requirements.

	integrate associated functions. • Be able to derive formulae for and calculate volumes of revolution about both the *x* and *y*-axes. • Be able to find volumes of revolution for functions given in parametric form.	
	FC2.6 Differential Equations • Be able to identify the form of first order differential equations that can be solved by an integrating factor and carry out the solution. • Be able to find general and particular solutions of differential equations of this form. • Be able to solve second order differential equations of the form $y'' + ay' + by = f(x)$ where $f(x)$ is a polynomial, exponential or trigonometric function. • Be able to find general and particular solutions of second order differential equations of this form. • Be able to use differential equations in modelling in kinematics and in other contexts. • Be able to solve the equation for simple harmonic motion $\ddot{x} = -\omega^2 x$ and relate the solution to the motion. • Be able to model damped oscillations using second order differential equations and interpret their solutions.	**FM1.2 Momentum & Impulse (Part 2)** • Be able to extend the definition of linear momentum and impulse to 2-D using vectors. • Be able to use the impulse-momentum principle in vector form, i.e. $\mathbf{I} = m\mathbf{v} - m\mathbf{u}$. **FM1.4 Elastic Strings & Springs** • Be able to investigate the ability of strings to stretch and springs to stretch and compress. • Be able to define the modulus of elasticity (λ), natural length (a) and extension (x). • Be able to use the above definitions to work out the tension in a stretched string or a stretched/compressed spring i.e. use Hooke's Law, $T = \frac{\lambda x}{a}$. • Be able to derive the elastic potential energy (EPE) from Hooke's Law by applying the work done in stretching a string/spring, i.e. EPE = $\frac{\lambda x^2}{2a}$. • Be able to calculate the tension in a string or spring when a system is held in equilibrium. • Be able to include EPE when using the work-energy principle. • Know the conditions for conservation of mechanical energy. • Be able to solve string/spring problems involving work and energy (i.e. KE, GPE and EPE).
4	Revision for final exams	**FM1.6 Elastic Collisions in Two Dimensions**

		• Understand that during an impact the impulse acts perpendicularly to the surface through the centre of the sphere. • Be able to apply Newton's (experimental) law of restitution in the direction of the impulse. • Appreciate that perpendicular to the impulse, the velocity component does not change. • Understand and be able to calculate an angle of deflection. • Be able to calculate the kinetic energy 'lost' in an impact. • Be able to work in speeds and angles or in velocity vectors (**i**, **j**). • Understand that, during a collision between two smooth spheres, total momentum is conserved and the impulse acts in the direction of the line of centres. • Be able to apply Newton's (experimental) law of restitution in the direction of the line of centres. • Appreciate that perpendicular to the line of centres, velocity components do not change. • Understand and be able to calculate an angle of deflection. • Be able to calculate the kinetic energy 'lost' in a collision. • Be able to work in speeds and angles or in velocity vectors (**i**, **j**).
5	Revision for final exams	Revision for final exams
6	Final exams (A Level)	
	Paper 1: Further Core Mathematics 25%, 1 hour 30 mins, 75 marks	Any pure content can be assessed on either paper
	Paper 2: Further Core Mathematics 25%, 1 hour 30 mins , 75 marks	
	Paper 3:	

	Decision Mathematics 1 25%, 1 hour 30 mins, 75 marks	
	Paper 4: Further Mechanics 1 25%, 1 hour 30 mins, 75 marks	